Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

portional to CM and BC. From C measure the distance CD=this third proportional. D is the required point.

For $BC^2=CM.CD$, or $BC^2=(AB-BC)CD$. $\therefore CD.AB=BC^2+BC.CD$.

$$CD.AB+BC.CD+CD^2=BC^2+2BC.CD+CD^2.$$

$$CD(AB+BC+CD)=(BC+CD)^2.$$

$$AD.CD=BD^2.$$

Also solved by S. A. Corey, J. R. Hitt, F. D. Posey, M. E. Graber, W. W. Landis, and G. W. Greenwood.

249. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of a square from three of its vertices, to find the side of the square.

250. Proposed by W. W. BEMAN, The University of Michigan.

Given the distances of a point in the plane of an equilateral triangle from the vertices; to find the side of triangle. [Perkins' *Geometry*, Olney's *Geometry*.]

I. Solution by F. D. POSEY, San Mateo, Calif.

Consider the general case, viz: Given the distances of a point in the plane of a regular n-gon to three consecutive vertices, to find the side of the n-gon.

Let the vertices be A, B, C in order, say clockwise, and P the given point. Let PA, PB, $PC\equiv a$, b, c, respectively. Let $\angle ABP=\alpha$, $\angle PBC=\beta$, taking these clockwise if P be without the angle ABC, and counter-clockwise if P be within. Call the side of the n-gon, x.

There are now two cases: (1) P within the angle ABC of the n-gon, (2) P without this angle. In the first case $\alpha+\beta=\frac{n-2}{n}\pi$. $\therefore \cos\beta=-\cos\frac{2\pi}{n}\cos\alpha+\sin\frac{2\pi}{n}\sin\alpha$. $\therefore \sin\alpha=\operatorname{cosec}\frac{2\pi}{n}\cos\beta+\cot\frac{2\pi}{n}\cos\alpha$.

In the second case $\alpha+\beta=2\pi-\frac{n-2}{n}\pi$. $\therefore \sin\alpha=-\operatorname{cosec}\frac{2\pi}{n}\cos\beta-\cot\frac{2\pi}{n}\cos\alpha$

Now $\cos\alpha=\dfrac{b^2+x^2-a^2}{2bx}$ (when c is between b and a we have $\cos(2\pi-\alpha)=\cos\alpha$), and $\cos\beta=\dfrac{b^2+x^2-c^2}{2bx}$ (when a is between b and c we have $\cos(2\pi-\beta)=\cos\beta$). In both cases (1) and (2), $\sin^2\alpha+\cos^2\alpha=(\operatorname{cosec}\frac{2\pi}{n}\cos\beta+\cot\frac{2\pi}{n}\cos\alpha)^2+\cos^2\alpha=1$, which equation after substituting the above values for $\cos\alpha$ and $\cos\beta$ reduces to:

$$\left[\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)^2+1\right]x^4+2\left\{\left(\cot\frac{2\pi}{n}+\operatorname{cosec}\frac{2\pi}{n}\right)\left[\cot\frac{2\pi}{n}(b^2-a^2)\right.\right.$$

$$\left.\left.+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]-a^2-b^2\right\}x^2+\left[\cot\frac{2\pi}{n}(b^2-a^2)+\operatorname{cosec}\frac{2\pi}{n}(b^2-c^2)\right]^2$$

$$+(b^2-a^2)^2=0.$$

In the case of the square, $n=4$. $\therefore \cot\frac{2\pi}{n}=0$, and $\operatorname{cosec}\frac{2\pi}{n}=1$.

The equation then becomes $2x^4-2(a^2+c^2)x^2+(b^2-a^2)^2+(b^2-c^2)^2=0$.

In the case of the equilateral triangle, $n=3$.

$\therefore \cot\frac{2\pi}{n}=-\frac{1}{\sqrt{3}}$, $\operatorname{cosec}\frac{2\pi}{n}=\frac{2}{\sqrt{3}}$, and we have $x^4-(a^2+b^2+c^2)x^2+a^4+b^4+c^4-a^2b^2-b^2c^2-c^2a^2=0$.

These equations may be solved as quadratics and by taking a unit length the expression for x may be constructed.

II. Solution by G. I. HOPKINS, Manchester, N. H.

Let AB, CD, and EF be the distances from the three vertices, and H the given vertical angle. Draw $\angle JKL=\angle H$ and make its sides both equal to AB, or any one of the three lengths. Then with J and L as centers, and CD and EF as radii fix the point N. Join NK. Then construct the isosceles $\triangle NKO$, NK being one of the equal sides, and vertical $\angle NKO=\angle H$.

$\therefore NOK$ is the required triangle. For, join JO. $JK=LK$, $NK=OK$, $\angle NKL=\angle JKO$. $\therefore \triangle JOK=\triangle NLK$. $\therefore NL=OJ$. $\therefore \triangle NKO$ is isosceles with vertical $\angle H$, and the three distances JK, JO, and JN are equal, respectively, to AB, CD, and EF.

If the triangle is equilateral, then $\angle H$ is two-thirds of a right angle, and we have the same construction.

Since a diagonal of a square divides it into two isosceles triangles whose vertical angles are 90°, therefore if $\angle H=90°$ we have the same construction and NK is a side of the required square.

Also solved by Henry Heaton, W. W. Landis, J. R. Hitt, M. E. Graber, and G. W. Greenwood.

CALCULUS.

189. Proposed by J. E. SANDERS, Hackney, Ohio.

Solve $d^2y/dx^2=-\beta^2(p+y)$, p and β being constants. The initial conditions are $y=0$ for $x=0$, l; $dy/dx=0$ for $x=l/2$. [Merriman's *Mechanics*, 9th Ed., 1903, §62.]

I. Solution by W. D. STAYTON, Student, Louisiana State University, Baton Rouge, La.; HOWARD L. STOKER, Lehigh University, and T. O. WHITAKER.

Multiplying by $2dy$ and integrating the resulting equation, we have

$$\left(\frac{dy}{dx}\right)^2=-2\beta^2y-\beta^2y^2+c_1 \quad\text{..........}(1).$$

Since $dy/dx=0$, for $y=\triangle$, $c_1=\beta^2\triangle^2+2\beta^2p\triangle$. Substituting this value of c_1 and extracting the square root, we have

$$dy/dx=\beta\sqrt{(-2py-y^2+\triangle^2+2\triangle p)}=\beta\sqrt{[(\triangle+p)^2-(y+p)^2]}.$$